Thank you for coming to the addition and subtraction workshop. We hope we have sent you two really important messages today.

Message 1 : The importance of knowing number facts to 20 by heart

Like multiplication and division facts are essential for being efficient at multiplying and dividing at a higher level, addition and subtraction facts are important for getting really good at adding and subtracting at a higher level. We want our children to know by heart facts like 3 + 7, 5 + 2, 14 + 5, 6 + 14 etc as well as 5 – 2, 6 – 3, 10 – 8, 18 – 3, and 20 – 17. Play the card game often to get really good at these.

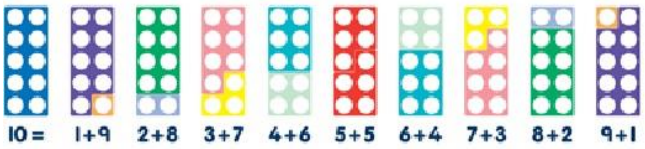
Message 2 : The importance of using manipulatives

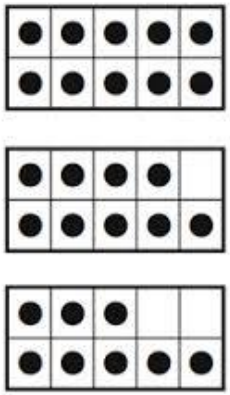
Manipulatives are very powerful in allowing children to 'see' and understand the mathematics in an idea. By asking children to use manipulatives to demonstrate results and prove their truth, we are developing their mathematical thinking at the deepest level and children are constructing solid foundations in their mathematical understanding. When children first encounter column methods for calculating, we use manipulatives alongside so that the children can explain each step in the procedure they are learning.

A manipulative is any piece of apparatus that helps children to 'see' the mathematics in an idea. Counters, dice, playing cards, straws, tens frames, numicon etc. are all manipulatives.

MANIPULATIVES TO SUPPORT THE DEVELOPMENT OF ADDITION AND SUBTRACTION CONCEPTS

Numicon: this is a resource we use to help children develop concepts about numbers. Children begin by getting to know the shapes and use them to investigate how different numbers can be made up. In this image, numicon is used to show pairs of numbers that total 10.





Tens Frames: Tens Frames are two-by-five rectangular frames into which counters are placed to illustrate numbers less than or equal to ten, and are therefore very useful for developing number sense within the context of ten. Various arrangements of counters on the tens frames can be used to prompt different mental images of numbers and different mental strategies for manipulating these numbers, all in association with the numbers' relationship to ten.

Double sided counters: these can be used to enable children to become familiar with number bonds, not only to 10, but for each number to 20. Children simply take a number of counters, for example 7, shake them, then throw them onto the table. The children will now see some counters which have landed red face up, and others which have landed yellow face up. They count the number of yellow faces and record. They count the number of red faces and record. They then use these number to create number sentences about the number 7. For example, 5 are red and 2 are yellow, so they may say 5 + 2 = 7 or 7 – 2 = 5

MANIPULATIVES TO SUPPORT THE DEVELOPMENT OF ADDITION AND SUBTRACTION CONCEPTS

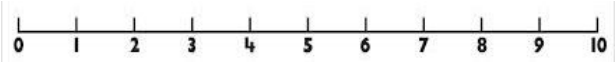
Straws: children begin by using straws to create, add and subtract numbers. When adding they count the straws and learn to bunch them in to groups of ten. It is an important step when young children see that ten ones is equal to one ten. This is the beginning of 'exchanging' and deepens their understanding of place value - how much each digit in a number is worth.

Dienes: children then move onto using Dienes, where numbers are represented as thousands, hundreds, tens and ones. Dienes are a step on from straws as a 10 in Dienes is represented by a single object, rather then 10 objects seen with the straws.

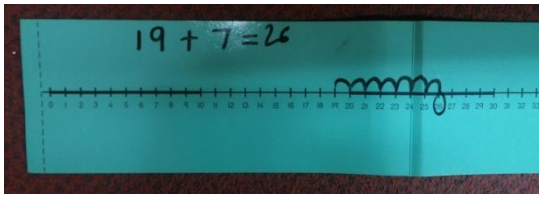
Place value counters: Once children are comfortable with Dienes, they move to a more abstract representation of number using these place value counters . Each counter has a different value, however, unlike Dienes, the size and the shape of each counter has no relation to its value.

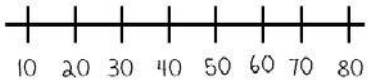
Place value cards: children create numbers using the place value cards. They place the different card together to create a number, say 173, then expand it to show the value of each digit. They can use this to write partitioned number sentences, 100 + 70 + 3 = 173, which facilitates understanding and recording when they move onto empty number line and column work.

THE DEVELOPMENT OF THE EMPTY NUMBER LINE (ENL) ADDITION Y2/Y3

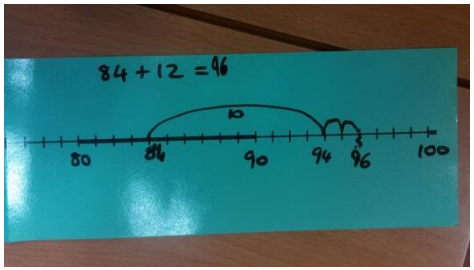


1. Number track: children begin by jumping forward on a number track, where the numbers are present.

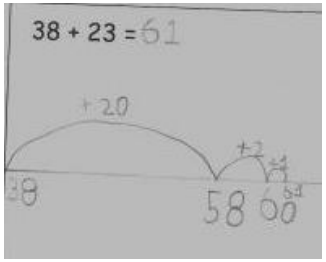


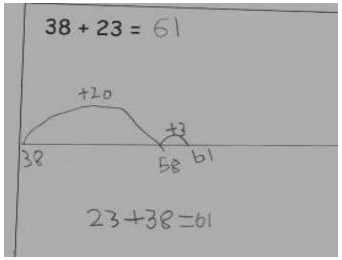


2. Partially calibrated number track: Here, only some numbers are labelled in the number track for children to use as a guide when adding by jumping on.

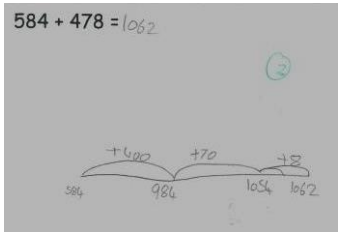


3. Empty number line (ENL): children then draw their own ENL. They begin by partitioning the smallest number so that they can jump tens and ones, making the process more efficient. They then place the largest number at the start of the number line and jump each part of the partitioned number. They label each jump to show each part of the addition, as well as the number they land on after each jump. The final number they land on is the answer. This is a more sophisticated and efficient process that relies on children being able to calculate from any number. We like it because it allows the children to be creative in their thinking and tells us a great deal about a child's understanding of number.

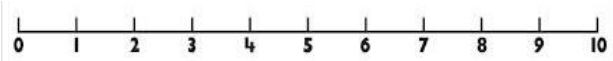




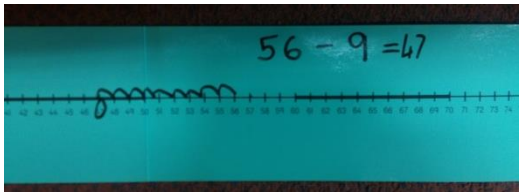
Children then progress to working with 3-digit numbers, partitioning them into hundreds, tens and ones in order to jump on the number line.

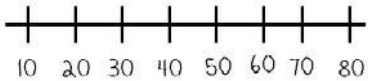


THE DEVELOPMENT OF THE EMPTY NUMBER LINE (ENL) SUBTRACTION Y2/Y3

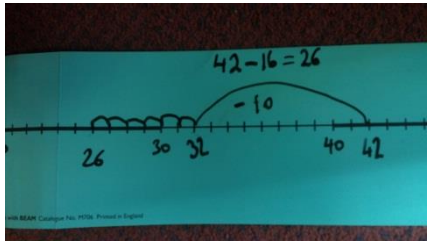


1. Number track: children begin by jumping back on a number track where the numbers are present.

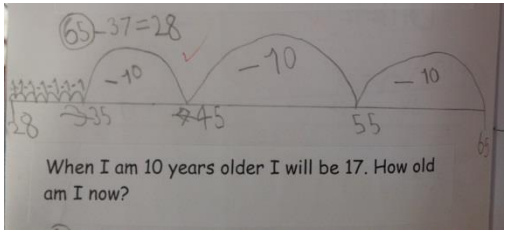


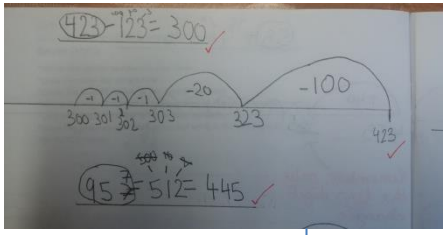


2. Partially calibrated number track: Here, only some numbers are labelled in the number track for children to use as a guide when subtracting by jumping back.

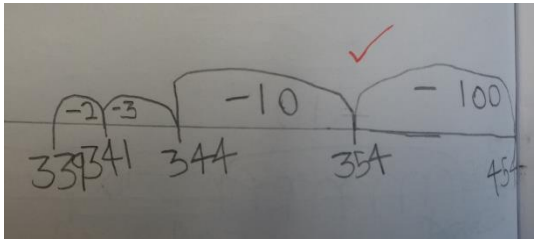


3. Empty number line (ENL): children then draw their own ENL. They begin by partitioning the smallest number so that they can jump tens and ones, making the process more efficient. They then place the largest number at the end of the number line and jump back each part of the partitioned number. They label each jump to show each part of the subtraction, as well as the number they land on after each jump. The final number that they land on is the answer. This allows children to begin subtracting in a more sophisticated, efficient way.



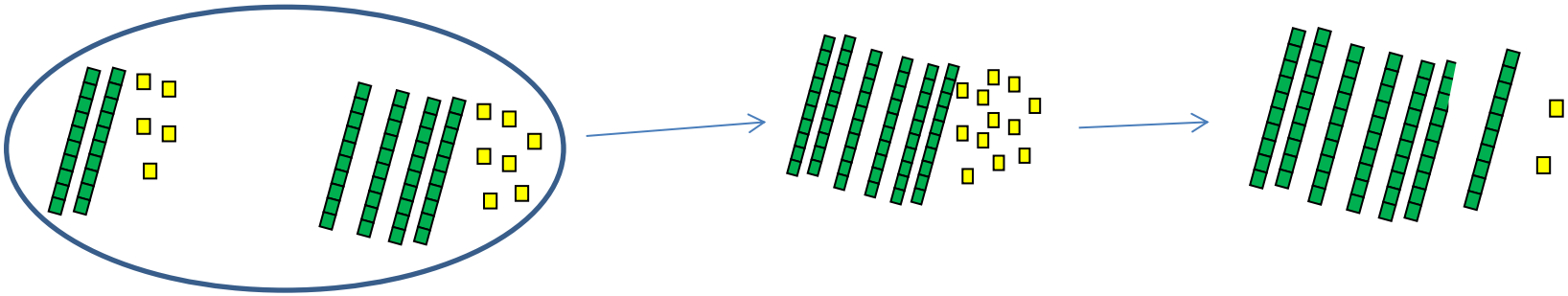


Children then progress to working with 3-digit numbers, partitioning them into hundreds, tens and ones. They group the ones rather than jumping each one individually.



<u>Children also use the ENL to find the difference between two numbers. This involves counting on, not back.</u>

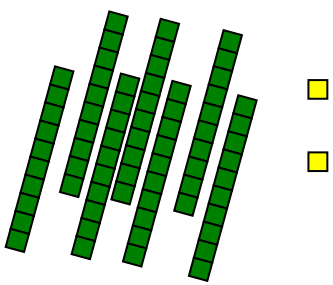
SUPPORTING STANDARD METHOD FOR ADDITION IN Y3 AND Y4

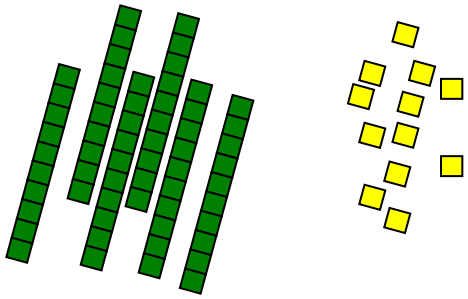


Above: Dienes for combining and exchanging

Above: place value counters are used on a calculation mat to combine and exchange. Note that the written representation runs alongside the manipulatives to reinforce understanding of the algorithm, and that the manipulatives are used to add from right to left to mirror the algorithm.

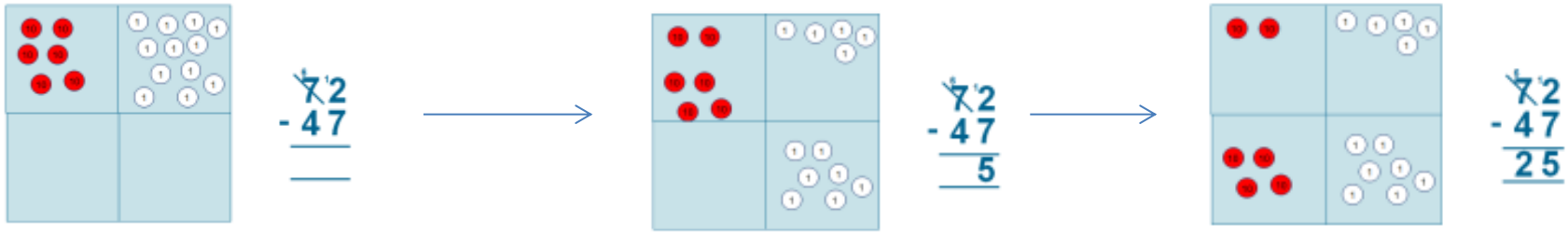
SUPPORTING STANDARD METHOD FOR SUBTRACTION IN Y3 AND Y4



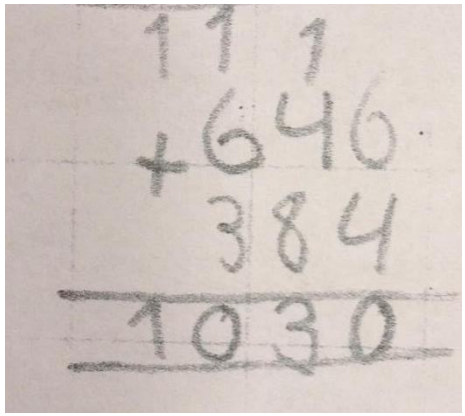


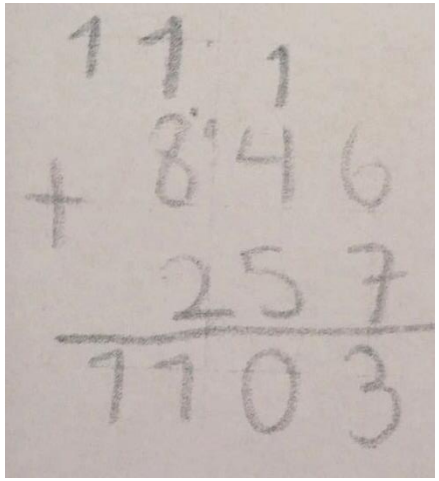
Far left: Dienes used to represent 72 in preparation for a subtraction.
Left: One Dienes 10 has been exchanged for 1s to allow for subtraction of any amount.

Below: place value counters are used on a calculation mat to represent the subtraction 72 -47.
Note how the calculation is carried out from right to left as in the written algorithm.



YEAR 5 – STANDARD METHOD FOR ADDITION

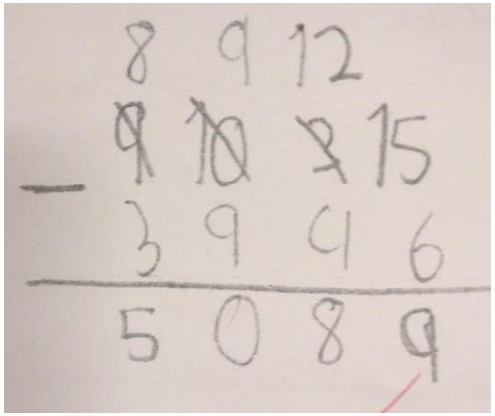




<u>646 + 384</u>

Starting from the right, she adds 6 ones to 4 ones giving 10 ones. She records this as a 0 in the unit place and mentally exchanges the ten ones as one ten so she records an extra ten in the tens place. Now she adds one ten to four tens to eight tens giving her 13 tens. Again, she splits the digits recording 3 in the tens place and thinks of the remaining ten 10s as one hundred which she records as an extra hundred in the hundreds place. Now she adds one hundred to six hundred to three hundred giving her 10 hundreds. She splits the digits, putting 0 in the hundreds place and thinks of the 10 hundreds as 1000 which she records as 1 in the thousands place. One thousand add 0 thousands is 1 thousand. So she records a 1 in the thousands place. Her answer is 1030.

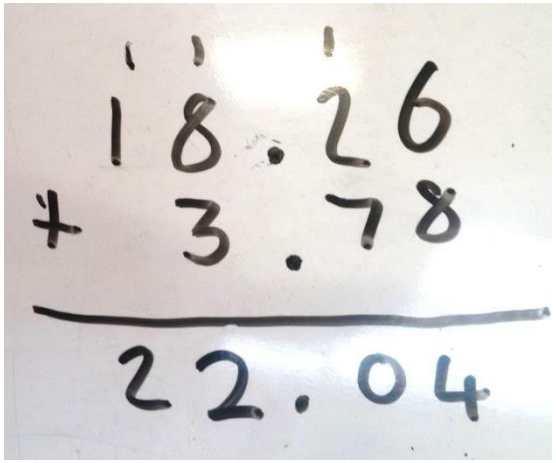
YEAR 5 – STANDARD METHOD FOR SUBTRACTION



<u>9035 - 3946</u>

Starting from the right, she cannot take 6 ones from 5 ones so she removes a ten from the tens place and thinks of it as 10 ones. Now she has 15 ones in the units. She subtracts 6 ones from 15 leaving her with 9 ones which she records below the line. Because she's already removed a ten from the tens place, she now only has 2 tens left (this is recorded as a 2 above the 3). She can't take 4 tens from 2 tens, so she looks to the hundreds hoping to take one from there and mentally exchange it for ten 10s. But there are no hundreds. So she looks to the thousands place where there are 9 thousands. She removes one thousand, leaving 8 thousand. She thinks of the 1000 she has removed as 10 hundreds and puts these in the hundreds column. Now she has some hundreds, she can remove one of them leaving 9 hundred, and exchange this for ten 10s. She now has 12 tens. She can take 4 tens from the 12 tens leaving 8 tens which she records below the line. Turning to the hundreds, she has 9 hundreds from which she needs to subtract 9 hundreds, leaving her with 0 hundreds which she records below the line as a 0 in the hundreds place. Finally she has 8 thousands from which she needs to subtract 3 thousand, leaving her with 5 thousand. Her answer is 5089.

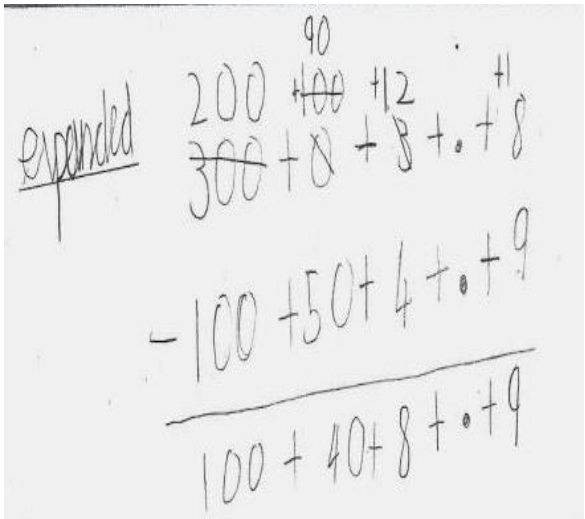
YEAR 6 – STANDARD METHOD FOR ADDITION



<u>18.26 + 3.78</u>

Starting from the right, she adds 6 hundredths to 8 hundredths giving 14 hundredths. She splits the digits, recording 4 in the hundredths place beneath the line and thinking of the 10 hundredths as one tenth which she records in the tenths column as a 1 above the 2 tenths. Now she adds one tenth to two tenths to seven tenths, giving her 10 tenths which she thinks of as one whole. She records this as a 1 above the 8. She records zero in the tenths place under the line. She adds 1 to 8 to 3, giving 12 ones. She records 2 ones under the line and thinks of the remaining ten ones as one ten. She records this as a 1 above the 1 ten that's already there. She has two tens altogether and records this as a 2 under the line. Her answer is 22.04.

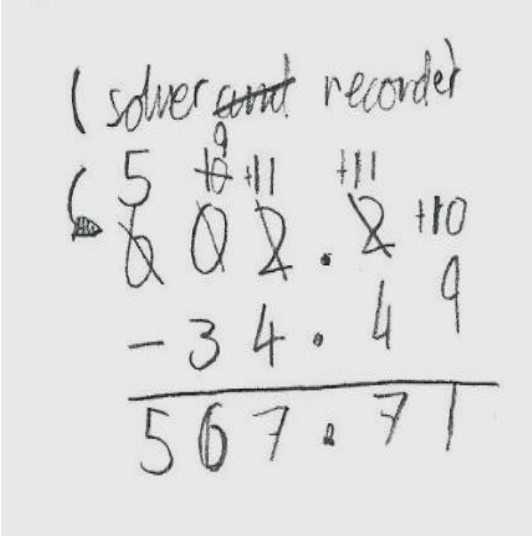
YEAR 6 – EXPANDED METHOD FOR SUBTRACTION



<u>303.8 – 154.9</u>

Using the expanded method, the children can clearly see the different parts of the number. 303.8 has been partitioned into 300, 0, 3 and 0.8 while 154.9 has been partitioned into 100, 50, 4 and 0.9. This allows the children to see the process more clearly. Starting from the right, the 0.9 is greater than 0.8 and so there needs to be an exchange. You can see the child has exchanged a unit and placed this into the tenths (he has recorded this as +1). Now he has 18 tenths, he can subtract 9 of them, leaving him with 9 tenths (or 0.9) which he records below the line as .+9. Then he has to take 4 ones from the 2 that remain on the top row. Again, the amount to be subtracted is greater than the amount 'in the pot' so the child does a further exchange. However, this time there are no tens to play with, so he takes 100 from the 300 and gives it to the tens, allowing him to take one ten (leaving 90) and give it to the ones, giving him a 10 and 2 (12) altogether in the ones. Now he can subtract 4 ones from 12 ones, leaving him with 8 ones which he records as +8. Now he has to take 50 away from 90, leaving 40 which he records below the line as +40, then he takes 100 from 200 leaving 100 which he records below the line. His answer is 148.9

YEARS 6 – STANDARD METHOD FOR SUBTRACTION (THIS IS AS HARD AS IT GETS!)



602.2 – 34.49

The standard method is the most efficient and we encourage its use once the children have shown a good understanding of the concepts behind each stage in the process.

To begin with the child records 602.2 as 602.20. Now he can exchange one tenth , turning it into ten hundredths (represented as +10 on the right hand side) and leaving just one tenth after the decimal point. Ten hundredths take away 9 hundredths leaves 1 hundredth which he records as a 1 under the line. He can't take 4 tenths from the one remaining tenth so he has to exchange one whole from the unit column, turning it into 10 tenths and combining it with one tenth that was already there, now recorded as +11. Eleven tenths take away 4 tenths is 7 tenths (recorded as 7 after the decimal point). Now moving to the unit column, again he doesn't have enough ones to subtract 4, so he has to exchange. This time, because there are no tens to make an exchange with, he exchanges from the hundreds, taking one hundred away leaving 5 hundreds and recording an extra 10 in the tens place (ten 10s is 100). Now he can take a ten from there, leaving 9 tens, and add it to the one unit he already had. This is recorded as +11 in the unit column. Now he can take 4 from 11 leaving 7 ones, 3 tens from 9 tens giving 6 tens and nothing to take away from 500 so he puts 5 in the hundred position beneath the line. His answer is 567.71. We would expect a child who understands this process to carry out the procedure within about 45 seconds.